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A Higher Dimensional Cosmic Domain Wall in Brans-Dicke Theory of Gravitation

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Abstract Five dimensional Kaluza-Klein Space-time is considered in the presence of thick domain walls in the scalar-tensor theory formulated by Brans and Dicke (Phys. Rev. 124:925, 1961). Exact cosmological model, in this theory, is presented with the help of special law of variation proposed by Berman (Nuovo Cim. B 74:182, 1983) for Hubble's parameter. Some physical and kinematical properties of the model are also discussed.

Keywords Domain walls · Brans-Dicke theory

1 Introduction

In recent years, there has been considerable interest in the study of early stages of evolution of the universe through cosmological models constructed in several theories of gravitation using different matter sources. At the very early stages of the universe, the topological defects such as cosmic strings, domain walls and monopoles are formed when the universe undergoes a series of phase transitions with discrete symmetry being spontaneously broken (Kibble et al. [1]). Of all these cosmological structures cosmic strings and domain walls have excited the most interest. In particular, domain walls have become more important in recent years from cosmological standpoint in view of a new scenario of galaxy formation proposed by Hill et al. [2]. According to them the formation of galaxies are due to domain walls produced during phase transitions after recombination of matter and radiation.

Vilenkin [3], Isper and Sikivie [4], Widrow [5], Goetz [6], Mukherjee [7] and Wang [8] are some of the authors who have investigated several aspects of domain walls in Einstein's theory of gravitation.

Brans-Dicke [9] theory of gravitation is the widely studied and best known alternative to Einstein's theory of gravitation. This theory accommodates both Mach's principle [10] and

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Dirac's large number hypothesis [11]. In this theory a scalar field ϕ is coupled to gravity with coupling constant ω. A detailed discussion of Brans-Dicke theory and Brans-Dicke cosmology is contained in the work of Singh and Rai [12].

Brans-Dicke [9] field equations for combined scalar and tensor fields are

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) - \phi^{-1}(\phi_{i;j} - g_{ij}\Box\phi) \tag{1}$$

$$\Box\phi = \phi_{;k}^{;k} = 8\pi\phi^{-1}T(3+2\omega)^{-1} \tag{2}$$

where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ is the Einstein tensor, T_{ij} is stress energy tensor of the matter, ω is the coupling constant and comma and semicolon denotes partial and covariant differentiation respectively.

Also the energy conservation equation

$$T^{ij}_{;j} = 0 \tag{3}$$

is a consequence of the field equations (1) and (2).

Rahaman and Bera [13], Rahaman [14, 15] Reddy and Rao [16], Chakraborty et al. [17] are some of the authors who have investigated domain walls in alternative theories of gravitation in four and five dimensions. Recently Adhov et al. [18] discussed four dimensional non-static domain walls in Brans-Dicke and Saez-Ballester [19] scalar-tensor theories of gravitation while Reddy et al. [20], very recently, investigated five dimensional domain walls in Saez-Ballester theory. The study of the thick domain walls and space-times associated with them are important due to their application in structure formation in the universe.

The fact that the cosmos at its early stage of evolution of the universe might have had a higher dimensional era has attracted many researchers to the field of higher dimensions. Solutions of field equations in higher dimensional space-time are believed to be of physical relevance possibly at the early times before the universe has undergone compactification transitions. Witten [21], Appelquist et al. [22], Chodos and Detweller [23] and Marciano [24] have studied several aspects of higher dimensional space-time.

In this paper, we investigate five dimensional Kaluza-Klein [25, 26] space-time in Brans-Dicke [9] theory of gravitation in the presence of thick domain walls. Domain walls in scalar-tensor theories of gravitation are getting special attention in cosmology due to their peculiar and interesting gravitational effects.

2 Basic Equations

We consider a five dimensional Kaluza-Klein space-time of the form

$$ds^2 = dt^2 - A^2(t)\left(dx^2 + dy^2 + dz^2\right) - B^2(t)d\psi^2 \tag{4}$$

where the fifth coordinate ψ is taken to be space-like.

The Einstein tensor components G^i_j for the space-time (4) may be written as [25, 26]

$$G^1_1 = 2\frac{A_{44}}{A} + \left(\frac{A_4}{A}\right)^2 + 2\frac{A_4B_4}{AB} + \frac{B_{44}}{B} = G^2_2 = G^3_3 \tag{5}$$

$$G^4_4 = 3\left(\frac{A_4}{A}\right)^2 + 3\frac{A_4B_4}{AB} \tag{6}$$

$$G_5^5 = 3\frac{A_{44}}{A} + 3\left(\frac{A_4}{A}\right)^2 \tag{7}$$

where the suffix 4 indicates differentiation with respect to t.

The domain wall is characterized by the energy momentum tensor [27]

$$T_{ij} = \rho(g_{ij} + \omega_i\omega_j) + p_1\omega_i\omega_j, \quad \omega^i\omega_j = -1 \tag{8}$$

where ρ is the energy density of the wall, p_1 is the pressure in the direction normal to the plane of the wall and ω_i is a unit space-like vector in the same direction.

With the help of (8) and (5)–(7) the field equations (1)–(3) for the metric (4) reduce to

$$2\frac{A_{44}}{A} + \frac{B_{44}}{B} + 2\frac{A_4B_4}{AB} + \left(\frac{A_4}{A}\right)^2 = -8\pi\phi^{-1}\rho - \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{A_4\phi_4}{A\phi} + \frac{\Box\phi}{\phi} \tag{9}$$

$$3\left(\frac{A_4}{A}\right)^2 + 3\frac{A_4B_4}{AB} = -8\pi\phi^{-1}\rho - \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{\phi_{44}}{\phi} + \frac{\Box\phi}{\phi} \tag{10}$$

$$3\frac{A_{44}}{A} + 3\left(\frac{B_4}{B}\right)^2 = 8\pi\phi^{-1}p_1 - \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{B_4\phi_4}{B\phi} + \frac{\Box\phi}{\phi} \tag{11}$$

$$\phi_{44} + \phi_4\left(3\frac{A_4}{A} + \frac{B_4}{B}\right) = \frac{8\pi\phi^{-1}}{3+2\omega}(4\rho - p_1) \tag{12}$$

$$\rho_4 - (\rho + p_1)\frac{B_4}{B} = 0 \tag{13}$$

Equations (9)–(13) give us four independent equations ((13) being consequence of the field equations) in five unknowns A, B, ρ, p_1, ϕ.

Hence to find a determinate solution we use

$$\rho = 4p_1 \tag{14}$$

which can be considered analogous to $\rho = 3p$ in general relativity for matter with disordered radiation in four-dimensional space-time.

Now the field equations (9)–(13) give us the following independent equations.

$$3\left(\frac{A_4}{A}\right)^2 + 3\frac{A_4B_4}{AB} = -8\pi\phi^{-1}\rho - \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{\phi_{44}}{\phi} \tag{15}$$

$$2\frac{A_{44}}{A} + \frac{B_{44}}{B} + 2\frac{A_4B_4}{AB} + \left(\frac{A_4}{A}\right)^2 = -8\pi\phi^{-1}\rho - \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{A_4\phi_4}{A\phi} \tag{16}$$

$$3\frac{A_{44}}{A} + 3\left(\frac{A_4}{A}\right)^2 = 8\pi\phi^{-1}p_1 - \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{B_4\phi_4}{B\phi} \tag{17}$$

$$\phi_{44} + \phi_4\left(3\frac{A_4}{A} + \frac{B_4}{B}\right) = 0 \tag{18}$$

3 Five Dimensional Cosmic Domain Wall

In this section we obtain cosmological model corresponding to thick domain wall in five dimensions in Brans-Dicke scalar-tensor theory of gravitation with the help of special law of variation for Hubble's parameter presented by Berman [28] that yields constant deceleration parameter models of the universe (the deceleration parameter measures the rate at which the expansion of the universe is slowing down). It may be noted here that most of the well known models of Einstein's theory and Brans-Dicke theory with curvature $K=0$, including inflationary models, are models with constant deceleration parameter. Berman and Gomide [29], Maharaj and Naidoo [30], Beesham [31], Johri and Desikan [32], Singh and Desikan [33] and Reddy et al. [34–36] have studied cosmological models with a constant deceleration parameter.

We consider constant deceleration parameter model defined by

$$q=-\left[\frac{RR_{44}}{(R_4)^2}\right]=\text{Constant} \tag{19}$$

where $R=(A^3B)^{1/3}$ is the over all scale factor of the universe. Here the constant is taken as negative (i.e. it is an accelerating model of the universe).

The solution of (19) is

$$R=(A^3B)^{1/3}=(at+b)^{1/1+q} \tag{20}$$

where $a\neq 0$ and b are constants of integration. This equation implies that the condition of expansion is $1+q>0$.

Also the field equations being highly non-linear we use a relation between the metric coefficients given by

$$B=A^n \tag{21}$$

where n is a constant.

Now with the help of (19)–(21) the field equations of Brans-Dicke theory admit the exact solution given by

$$A=(at+b)^{\frac{3}{(1+q)(n+3)}} \tag{22}$$

$$B=(at+b)^{\frac{3n}{(1+q)(n+3)}} \tag{23}$$

$$\phi=\left(\frac{k_1}{a}\right)\left(\frac{1+q}{q-2}\right)(at+b)^{\frac{q-2}{1+q}}+\phi_0 \tag{24}$$

$$\begin{aligned}8\pi p_1&=8\pi(4\rho)\\&=\left(\frac{k_1a}{q-2}\right)(at+b)^{-(\frac{q+4}{1+q})}\left[\frac{(q-2)(n+3)\{\omega(q-2)(n+3)-6\}+108}{2(1+q)(n+3)^2}\right]\end{aligned} \tag{25}$$

where $(1+q)$, $(n+3)$ and $(q-2)>0$ and the constants ω, q and n satisfy a relation of the form

$$(q-2)(n+3)[\omega(q-2)(n+3)+6(n-4)]-36(2n+3)=0 \tag{26}$$

Now through a proper choice of coordinates and constants we can write, the five dimensional cosmic domain wall model in Brans-Dicke theory, as

$$ds^2=dT^2-T^{\frac{6}{(1+q)(n+3)}}(dX^2+dY^2+dZ^2)-T^{\frac{6n}{(1+q)(n+3)}}d\psi^2 \tag{27}$$

4 Physical Properties of the Model

Here we discuss some physical and kinematical properties of the cosmic domain wall given by (27). This represents a five dimensional non-static cosmic domain wall model which has no initial singularity.

The physical quantities which are important in cosmology are proper volume V^3, the expansion scalar θ, shear scalar σ^2, Hubble's parameter H, density ρ, pressure p_1 and the scalar field ϕ. They have the following expressions for the model given by (27):

$$V^3 = \sqrt{-g} = T^{\frac{3}{1+q}} \tag{28}$$

$$\theta = \frac{1}{3}\omega^i_{;i} = \frac{1}{(1+q)T} \tag{29}$$

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{1}{6[(1+q)T]^2} \tag{30}$$

$$H = \frac{R_4}{R} = \frac{1}{(1+q)T} \tag{31}$$

$$8\pi p_1 = 8\pi(4\rho) = \left[\frac{(q-2)(n+3)\{\omega(q-2)(n+3)-6\}+108}{2(1+q)(n+3)^2}\right] T^{-(\frac{q+4}{1+q})}$$

$$\phi = \left(\frac{1+q}{q-2}\right) T^{-(\frac{q-2}{1+q})}$$

It may be observed that at initial moment when $T = 0$, the spatial volume will be zero while the energy density ρ, pressure p_1 and the scalar field ϕ diverge. Also when $T \to 0$, the expansion scalar θ, shear scalar σ^2 and Hubble's parameter H tend to infinity.

For large values of T (i.e. $T \to \infty$) we observe that spatial volume, expansion scalar θ, shear scalar σ^2, Hubble's parameter H, pressure p_1 energy density ρ and the scalar field ϕ become zero.

Also $\lim_{T\to\infty}(\frac{\sigma}{\theta})^2 \neq 0$ and hence the model does not approach isotropy.

In this model, the particle horizon exists because

$$\int_{T_0}^{T} \frac{dT}{V(T)} = \left(\frac{q+1}{q}\right)\left(T^{\frac{q}{q+1}}\right)_{T_0}^{T} \tag{32}$$

is a convergent integral.

5 Conclusions

Here we have obtained a non-static cosmic domain wall with the help of a five-dimensional Kaluza-Klein space-time in Brans-Dicke theory using the special law of variation proposed by Berman [28] for Hubble's parameter. We have also used a relation between the metric potentials and an equation of state corresponding to pressure and energy density of the thick domain wall. Thick domain walls and the space-times associated with them have cosmological interest due to their important applications in structure formation of the universe. Also, since scalar fields have considerable effects in the early stages of evolution of the universe, the cosmic domain walls associated with them will be useful for a better understanding of Brans-Dicke cosmology.

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